Nonleptonic $B \to D^{(*)} D_{sJ}^{(*)}$ decays and the nature of the orbitally excited charmed-strange mesons

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The Belle Collaboration has recently reported a study of the decays $B \to D_{s1}(2536)^+ \bar{D}^{(*)}$ and has given also estimates of relevant ratios between branching fractions of decays $B \to D^{(*)} D_{sJ}^{(*)}$ providing important information to check the structure of the $D_{s0}^*(2317)$, $D_{s1}(2460)$ and $D_{s1}(2536)$ mesons. The disagreement between experimental data and Heavy Quark Symmetry has been used as an indication that $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons could have a more complex structure than the canonical $c\bar{s}$ one. We analyze these ratios within the framework of a constituent quark model, which allows us to incorporate the effects given by finite c-quark mass corrections. Our findings are that while the $D_{s1}(2460)$ meson could have a sizable non-$q\bar{q}$ component, the $D_{s0}^*(2317)$ and $D_{s1}(2536)$ mesons seem to be well described by a pure $q\bar{q}$ structure.

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I. INTRODUCTION

B-factories, through e^+e^- collisions, tuned to different bottomonium energy resonances, have become a source of data on heavy hadrons. Bottomonium states decay most of the cases to $B\bar{B}$ pairs, and these B mesons decay subsequently into charmed and charmless hadrons via the weak interaction.

Nonleptonic decays of B mesons, described at quark level by an effective four-quark interaction $\bar{b} \to \bar{c}c\bar{s}$, have been used to search for new charmonium and charmed-strange mesons and to study their properties in detail. Within the charmed-strange sector, the BaBar Collaboration found, in the inclusive $D_s^+\pi^0$ invariant mass distribution from e^+e^- annihilation data, the narrow state $D_{s0}^*(2317)$ [1]. The CLEO Collaboration, aiming to confirm the previous state, observed its doublet partner $D_{s1}(2460)$ in the $D_s^{*+}\pi^0$ final state [2]. However, the properties of these states were not well known until the Belle Collaboration observed the $B \to \bar{D}D_{s0}^*(2317)$ and $B \to \bar{D}D_{s1}(2460)$ decays [3].

First observations of the $B \to \bar{D}^{(*)}D_{s1}(2536)$ decay modes have been reported by the BaBar Collaboration [4, 5] and an upper limit on the decay $B^0 \to D^{*-}D_{s1}(2536)^+$ was also obtained by the Belle Collaboration [6]. The most recent analysis of the production of $D_{s1}(2536)^+$ in double charmed B meson decays has been reported by the Belle Collaboration in Ref. [7]. Using the latest measurements of the $B \to D^{(*)}D_{sJ}^{(*)}$ branching fractions [8] they calculate the ratios

$$R_{D0} = \frac{\mathcal{B}(B \to DD_{s0}^*(2317))}{\mathcal{B}(B \to DD_s)} = 0.10 \pm 0.03,$$
$$R_{D^*0} = \frac{\mathcal{B}(B \to D^*D_{s0}^*(2317))}{\mathcal{B}(B \to D^*D_s)} = 0.15 \pm 0.06,$$

$$\begin{aligned} R_{D1} &= \frac{\mathcal{B}(B \to DD_{s1}(2460))}{\mathcal{B}(B \to DD_s^*)} = 0.44 \pm 0.11, \\ R_{D^*1} &= \frac{\mathcal{B}(B \to D^*D_{s1}(2460))}{\mathcal{B}(B \to D^*D_s^*)} = 0.58 \pm 0.12. \end{aligned} \quad (1)$$

In addition, the same ratios are calculated for $B \to D^{(*)}D_{s1}(2536)^+$ decays, using combined results by the BaBar [5] and Belle [7] Collaborations

$$\begin{aligned} R_{D1'} &= \frac{\mathcal{B}(B \to DD_{s1}(2536))}{\mathcal{B}(B \to DD_s^*)} = 0.049 \pm 0.010, \\ R_{D^*1'} &= \frac{\mathcal{B}(B \to D^*D_{s1}(2536))}{\mathcal{B}(B \to D^*D_s^*)} = 0.044 \pm 0.010. \end{aligned} \quad (2)$$

¿From a theoretical point of view, this kind of decays can be described using the factorization approximation [9]. This amounts to evaluate the matrix element which describes the $B \to D^{(*)}D_{sJ}^{(*)}$ weak decay process as a product of two matrix elements, one for the B weak transition into the $D^{(*)}$ meson and the second one for the weak creation of the $c\bar{s}$ pair which makes the $D_{sJ}^{(*)}$ meson. The latter matrix element is proportional to the corresponding $D_{sJ}^{(*)}$ meson decay constant and therefore these processes can give information about the D_{sJ} structure.

The $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states belong to the doublet $j_q^P = \frac{1}{2}^+$ predicted by Heavy Quark Symmetry (HQS). They have surprisingly light masses and are located below DK and D^*K thresholds, respectively. This implies that these states are narrow. Neither quark models nor Lattice QCD calculations [10] can give a satisfactory explanation of this doublet in the charmed and charmed-strange sectors. This fact has stimulated a fruitful line of research, suggesting that their structure is much richer than what one might guess assuming the $q\bar{q}$ picture, which has proved quite successful in

other sectors. Some authors have suggested a $qq\bar{q}\bar{q}$ structure [11, 12] or mixing between the usual $q\bar{q}$ structure and four quark [13]. Also there have been suggestions that these states might be molecular states. However, one must mention that the $\mathcal{B}(B \to D_{s0}^*(2317)K)$ is the same order of magnitude as $\mathcal{B}(B \to D_s K)$ and at least a factor of 2 larger than the $\mathcal{B}(B \to D_{s1}(2460)K)$ branching fraction [14], in contrast with the naive expectation that decays within the same spin doublet would have similar rates. This fact suggests that the $D_{s0}^*(2317)$ meson may be a conventional 0^+ $c\bar{s}$ state and the $D_{s1}(2460)$ has a more complex structure.

In any case the study of nonleptonic decays can help to shed light on the structure of the P-wave open charm mesons

The $D_{sJ}^{(*)}$ meson decay constants are not known experimentally except for the ground state, D_s, which has been measured by different collaborations. Another way to study $D_{sJ}^{(*)}$ mesons that does not rely on the knowledge of their decay constants is through the decays $B_s \to D_{sJ}^{(*)}M$ where M is a meson with a well known decay constant. However, the experimental study of these processes is currently difficult leaving, for the time being, the $B \to D^{(*)}D_{sJ}^{(*)}$ decay processes as our best option to study $D_{sJ}^{(*)}$ meson properties.

According to Ref. [15], within the factorization approximation and in the heavy quark limit, the ratios R_{D0} and R_{D1} can be written as

$$\begin{aligned} R_{D0} = R_{D^*0} &= \left|\frac{f_{D_{s0}^*(2317)}}{f_{D_s}}\right|^2, \\ R_{D1} = R_{D^*1} &= \left|\frac{f_{D_{s1}(2460)}}{f_{D_s^*}}\right|^2, \end{aligned} \tag{3}$$

where the phase space effects are neglected because they are subleading in the heavy quark expansion. Now, in the heavy quark limit one has $f_{D_{s0}^*} = f_{D_{s1}}$ and $f_{D_s} = f_{D_s^*}$ and so one would predict $R_{D0} \approx R_{D1}$. Moreover, there are several estimates of the decay constants, always in the heavy quark limit [16–18], that predict for P-wave $j_q = 1/2$ states similar decay constants as for the ground state mesons (i.e. D_s and D_s^*), and very small decay constants for P-wave, $j_q = 3/2$ states. Then, these approximations lead to ratios of order one for $D_{s0}^*(2317)$ and $D_{s1}(2460)$, in strong disagreement with experiment.

Leaving aside that the factorization approximation has been recently analyzed in Refs. [19–21] finding that it works well in these kind of processes, we will concentrate in the influence of the effect of the finite c-quark mass in the theoretical predictions. As found in Ref. [22], $1/m_Q$ contributions could give large corrections to various quantities describing $B \to D^{**}$ transitions and we expect they could also play an important role in this case.

We work within the framework of the constituent quark model described in Ref. [23] and which is widely used in hadronic spectroscopy [23, 24]. It has recently been applied to mesons containing heavy quarks in Refs. [25, 26]. It has also been used successfully in the description of semileptonic $B \to D^{**}$ and $B_s \to D_s^{**}$ decays in Ref. [27], which provides us with confidence that the model describes well the weak decay transitions $B \to D^{(*)}$ which is one of the terms in the calculation of $B \to D^{(*)}D_{sJ}^{(*)}$ decays.

The paper is organized as follows: In Sec. II, we introduce the constituent quark model and discuss its predictions in the charmed and charmed-strange sectors. Sec. III is devoted to explain the theoretical framework through which we calculate the $B \to D^{(*)}D_{sJ}^{(*)}$ decays. Finally, we present our results in Sec. IV and give the conclusions in Sec. V.

II. CONSTITUENT QUARK MODEL

Constituent light quark masses and Goldstone-boson exchanges coming from the spontaneous chiral symmetry breaking of the QCD Lagrangian, together with the perturbative one-gluon exchange (OGE) and the nonperturbative confining interaction are the main pieces of potential models. Using this idea, Vijande *et al.* [23] developed a model of the quark-quark interaction which is able to describe meson phenomenology from the light to the heavy quark sector.

A consistent description of light, strange and heavy mesons requires an effective scale-dependent strong coupling constant. We use the frozen coupling constant of Ref. [23]

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\left(\frac{\mu_{ij}^2+\mu_0^2}{\Lambda_0^2}\right)}, \tag{4}$$

where μ_{ij} is the reduced mass of the $q\bar{q}$ pair, and α_0, μ_0 and Λ_0 are parameters of the model determined by a global fit to all meson spectrum.

In the heavy quark sector chiral symmetry is explicitly broken and Goldstone-boson exchanges do not appear. Thus, OGE and confinement are the only interactions remaining.

The one-gluon exchange potential is generated from the vertex Lagrangian

$$\mathcal{L}_{qqg} = i\sqrt{4\pi\alpha_s}\bar{\psi}\gamma_\mu G_c^\mu \lambda^c \psi, \tag{5}$$

where λ^c are the $SU(3)$ color matrices and G_c^μ is the gluon field. The resulting potential contains central, tensor and spin-orbit contributions given by

$$
\begin{aligned}
V_{\rm OGE}^{\rm C}(\vec{r}_{ij}) &= \frac{1}{4}\alpha_s(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}}-\frac{1}{6m_im_j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}r_0^2(\mu)}\right],\\
V_{\rm OGE}^{\rm T}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_im_j}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3}-\frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}\left(\frac{1}{r_{ij}^2}+\frac{1}{3r_g^2(\mu)}+\frac{1}{r_{ij}r_g(\mu)}\right)\right]S_{ij},\\
V_{\rm OGE}^{\rm SO}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_i^2m_j^2}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3}-\frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3}\left(1+\frac{r_{ij}}{r_g(\mu)}\right)\right]\times\\
&\qquad\times\left[((m_i+m_j)^2+2m_im_j)(\vec{S}_+\cdot\vec{L})+(m_j^2-m_i^2)(\vec{S}_-\cdot\vec{L})\right],
\end{aligned}
\tag{6}
$$

where $r_0(\mu_{ij})=\hat{r}_0\frac{\mu_{nn}}{\mu_{ij}}$ and $r_g(\mu_{ij})=\hat{r}_g\frac{\mu_{nn}}{\mu_{ij}}$ are regulators. Note the contact term of the central part of the one-gluon exchange potential has been regularized as follows

$$
\delta(\vec{r}_{ij})\sim\frac{1}{4\pi r_0^2}\frac{e^{-r_{ij}/r_0}}{r_{ij}}.
\tag{7}
$$

Although there is no analytical proof, it is a general belief that confinement emerges from the force between the gluon color charges. When two quarks are separated, due to the non-Abelian character of the theory, the gluon fields self-interact forming color strings which bring the quarks together.

In a pure gluon gauge theory the potential energy of the $q\bar{q}$ pair grows linearly with the quark-antiquark distance. However, in full QCD the presence of sea quarks may soften the linear potential, due to the screening of the color charges, and eventually leads to the breaking of the string. We incorporate this feature in our confinement potential as

$$
\begin{aligned}
V_{\rm CON}^{\rm C}(\vec{r}_{ij}) &= \left[-a_c(1-e^{-\mu_c r_{ij}})+\Delta\right](\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c),\\
V_{\rm CON}^{\rm SO}(\vec{r}_{ij}) &= -(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\frac{a_c\mu_c e^{-\mu_c r_{ij}}}{4m_i^2m_j^2r_{ij}}\times\\
&\times\Big[((m_i^2+m_j^2)(1-2a_s)+4m_im_j(1-a_s))(\vec{S}_+\cdot\vec{L})\\
&\quad+(m_j^2-m_i^2)(1-2a_s)(\vec{S}_-\cdot\vec{L})\Big],
\end{aligned}
\tag{8}
$$

where a_s controls the mixture between the scalar and vector Lorentz structures of the confinement. At short distances this potential presents a linear behavior with an effective confinement strength, $\sigma=-a_c\,\mu_c\,(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)$, while it becomes constant at large distances. This type of potential shows a threshold defined by

$$
V_{\rm thr}=\{-a_c+\Delta\}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c).
\tag{9}
$$

No $q\bar{q}$ bound states can be found for energies higher than this threshold. The system suffers a transition from a color string configuration between two static color sources into a pair of static mesons due to the breaking of the color flux-tube and the most favored subsequent decay into hadrons.

Quark masses	m_n (MeV)	313
	m_s (MeV)	555
	m_c (MeV)	1763
	m_b (MeV)	5110
OGE	α_0	2.118
	Λ_0 (fm^{-1})	0.113
	μ_0 (MeV)	36.976
	$\hat{r}_0$ (fm)	0.181
	$\hat{r}_g$ (fm)	0.259
Confinement	a_c (MeV)	507.4
	μ_c (fm^{-1})	0.576
	Δ (MeV)	184.432
	a_s	0.81

TABLE I. Quark model parameters.

Among the different methods to solve the Schrödinger equation in order to find the quark-antiquark bound states, we use the Gaussian Expansion Method [28] which provides enough accuracy and it simplifies the subsequent evaluation of the decay amplitude matrix elements.

This procedure provides the radial wave function solution of the Schrödinger equation as an expansion in terms of basis functions

$$
R_\alpha(r)=\sum_{n=1}^{n_{max}}c_n^\alpha\phi_{nl}^G(r),
\tag{10}
$$

where α refers to the channel quantum numbers. The coefficients, c_n^α, and the eigenvalue, E, are determined from the Rayleigh-Ritz variational principle

$$
\sum_{n=1}^{n_{max}}\left[(T_{n'n}^\alpha-EN_{n'n}^\alpha)\,c_n^\alpha+\sum_{\alpha'}\ V_{n'n}^{\alpha\alpha'}c_n^{\alpha'}=0\right],
\tag{11}
$$

where $T_{n'n}^\alpha$, $N_{n'n}^\alpha$ and $V_{n'n}^{\alpha\alpha'}$ are the matrix elements of the kinetic energy, the normalization and the potential, respectively. $T_{n'n}^\alpha$ and $N_{n'n}^\alpha$ are diagonal, whereas the mixing between different channels is given by $V_{n'n}^{\alpha\alpha'}$.

Following Ref. [28], we employ Gaussian trial functions with ranges in geometric progression. This enables the optimization of ranges employing a small number of free parameters. Moreover, the geometric progression is dense at short distances, so that it enables the description of the dynamics mediated by short range potentials. The

fast damping of the gaussian tail does not represent an issue, since we can choose the maximal range much longer than the hadronic size.

Model parameters fitted over the whole meson spectra [23, 25] are shown in Table I.

The model described above is not able to reproduce the spectrum of the P-wave charmed and charmed-strange mesons. The inconsistency with experiment is mainly due to the fact that the mass splittings between the $D_{s0}^*(2317)$, $D_{s1}(2460)$ and $D_{s1}(2536)$ mesons are not well reproduced. The same problem appears in Lattice QCD calculations or other quark models [10].

In order to improve these mass splittings we follow the proposal of Ref. [29] and include one-loop corrections to the OGE potential as derived by Gupta *et al.* [30]. This corrections shows a spin-dependent term which affects only mesons with different flavor quarks.

The net result is a quark-antiquark interaction that can be written as:

$$V(\vec{r}_{ij}) = V_{\rm OGE}(\vec{r}_{ij}) + V_{\rm CON}(\vec{r}_{ij}) + V_{\rm OGE}^{\rm 1-loop}(\vec{r}_{ij}), \quad (12)$$

where $V_{\rm OGE}$ and $V_{\rm CON}$ were defined before and are treated non-perturbatively. $V_{\rm OGE}^{\rm 1-loop}$ is the one-loop correction to OGE potential which is treated perturbatively. As in the case of $V_{\rm OGE}$ and $V_{\rm CON}$, $V_{\rm OGE}^{\rm 1-loop}$ contains central, tensor and spin-orbit contributions, given by [29]

$$\begin{aligned}
&V_{OGE}^{\rm 1-loop,C}(\vec{r}_{ij}) = 0, \\
&V_{OGE}^{\rm 1-loop,T}(\vec{r}_{ij}) = \frac{C_F}{4\pi}\frac{\alpha_s^2}{m_i m_j}\frac{1}{r^3}S_{ij}\left[\frac{b_0}{2}\left(\ln(\mu r_{ij}) + \gamma_E - \frac{4}{3}\right) + \frac{5}{12}b_0 - \frac{2}{3}C_A \right. \\
&\qquad\qquad \left. + \frac{1}{2}\left(C_A + 2C_F - 2C_A\left(\ln(\sqrt{m_i m_j}\, r_{ij}) + \gamma_E - \frac{4}{3}\right)\right)\right], \\
&V_{OGE}^{\rm 1-loop,SO}(\vec{r}_{ij}) = \frac{C_F}{4\pi}\frac{\alpha_s^2}{m_i^2 m_j^2}\frac{1}{r^3}\times \\
&\times\Bigg\{(\vec{S}_+\cdot\vec{L})\Big[\left((m_i+m_j)^2 + 2m_i m_j\right)\left(C_F + C_A - C_A\left(\ln(\sqrt{m_i m_j}\, r_{ij}) + \gamma_E\right)\right) \\
&\qquad + 4m_i m_j\left(\frac{b_0}{2}\left(\ln(\mu r_{ij}) + \gamma_E\right) - \frac{1}{12}b_0 - \frac{1}{2}C_F - \frac{7}{6}C_A + \frac{C_A}{2}\left(\ln(\sqrt{m_i m_j}\, r_{ij}) + \gamma_E\right)\right) \\
&\qquad + \frac{1}{2}(m_j^2 - m_i^2)C_A\ln\left(\frac{m_j}{m_i}\right)\Big] \\
&\quad + (\vec{S}_-\cdot\vec{L})\Big[(m_j^2 - m_i^2)\left(C_F + C_A - C_A\left(\ln(\sqrt{m_i m_j}\, r_{ij}) + \gamma_E\right)\right) \\
&\qquad + \frac{1}{2}(m_i+m_j)^2 C_A\ln\left(\frac{m_j}{m_i}\right)\Big]\Bigg\},
\end{aligned} \quad (13)$$

where $C_F = 4/3$, $C_A = 3$, $b_0 = 9$, $\gamma_E = 0.5772$ and the scale $\mu \sim 1\,\text{GeV}$.

Table II shows the masses of well established charmed and charmed-strange mesons predicted by the constituent quark model and those including one-loop corrections to the OGE potential.

The masses predicted for the 0^- and 1^- states agree with the experimental measurements in both sectors. These states can be identified with the members of the lowest lying $j_q^P = \frac{1}{2}^-$ doublet predicted by HQS.

The doublet $j_q^P = \frac{3}{2}^+$, which corresponds to the 2^+ state and one of the low lying 1^+ states, is in reasonable agreement with experiment.

The charmed and charmed-strange 0^+ states are sensitive to the one-loop corrections of the OGE potential which bring their masses closer to experiment. However, the spin dependent corrections to the OGE potential, are not enough to solve the puzzle in the 1^+ sector. A possible explanation for the low mass of this state has been proposed in Ref. [26] were the 1^+ $c\bar{s}$ states are coupled to a $c\bar{s}n\bar{n}$ tetraquark structure. One finds that the $J^P = 1^+$ $D_{s1}(2460)$ has an important non-$q\bar{q}$ contribution whereas the $J^P = 1^+$ $D_{s1}(2536)$ is almost a pure $q\bar{q}$ state.

III. NONLEPTONIC $B \to D^{(*)}D_{sJ}^{(*)}$ DECAYS

We give account of the nonleptonic decays $B \to D^{(*)}D_{sJ}^{(*)}$ with $D^{(*)}$ the D or D^* mesons, and $D_{sJ}^{(*)}$ the mesons $D_{s0}^*(2317)$, $D_{s1}(2460)$ and $D_{s1}(2536)$. These decay modes involve a $b \to c$ transition at the quark level,

	Charmed					
	$j_q^P = 1/2^-$		$j_q^P = 1/2^+$		$j_q^P = 3/2^+$	
	0^-	1^-	0^+	1^+	1^+	2^+
This work (α_s)	1896	2017	2516	2596	2466	2513
This work (α_s^2)	1896	2014	2362	2535	2499	2544
Exp.	1867.7 ± 0.3	2010.25 ± 0.14	2403 ± 38	2427 ± 36	2423.4 ± 3.1	2460.1 ± 4.4
	Charmed-strange					
	$j_q^P = 1/2^-$		$j_q^P = 1/2^+$		$j_q^P = 3/2^+$	
	0^-	1^-	0^+	1^+	1^+	2^+
This work (α_s)	1984	2110	2510	2593	2554	2591
This work (α_s^2)	1984	2104	2383	2570	2560	2609
Exp.	1969.0 ± 1.4	2112.3 ± 0.5	2318.0 ± 1.0	2459.6 ± 0.9	2535.12 ± 0.25	2572.6 ± 0.9

TABLE II. Masses of well established charmed and charmed-strange mesons predicted by the constituent quark model (α_s) and those including one-loop corrections to the OGE potential (α_s^2).

governed by the effective Hamiltonian [9, 31, 32]

$$H_{\text{eff.}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} \left[C_1(\mu')\, Q_1^{cb} + C_2(\mu')\, Q_2^{cb} \right] + h.c. \right\}, \tag{14}$$

in which penguin operators have been neglected. In Eq. (14), $C_1(\mu')$ and $C_2(\mu')$ are scale-dependent Wilson coefficients, being $\mu' \simeq m_b$ the appropriate energy scale in this case. The Q_1^{cb} and Q_2^{cb} are local four-quark operators given by

$$\begin{aligned} Q_1^{cb} &= V_{cs}^* \left[\bar\psi_c(0)\gamma_\mu(\mathcal{I}-\gamma_5)\psi_b(0) \right] \left[\bar\psi_s(0)\gamma^\mu(\mathcal{I}-\gamma_5)\psi_c(0) \right], \\ Q_2^{cb} &= V_{cs}^* \left[\bar\psi_s(0)\gamma_\mu(\mathcal{I}-\gamma_5)\psi_b(0) \right] \left[\bar\psi_c(0)\gamma^\mu(\mathcal{I}-\gamma_5)\psi_c(0) \right]. \end{aligned} \tag{15}$$

We show in Fig. 1 the schematic representation of the $B^- \to D^{(*)0} D_{sJ}^{(*)-}$ decay given by the two local four-quark operators, diagram d_1 for Q_1^{cb} and diagram d_2 for Q_2^{cb}. Factorization approximation is implicit in diagram d_1, which amounts to evaluate the hadron matrix element of the effective Hamiltonian as a product of quark-current matrix elements. Fierz reordering of diagram d_2 leads to the same contribution but for a colour factor. The full transition amplitude can be evaluated with the Q_1^{cb} part of the Hamiltonian but with an effective coupling given by

$$a_1(\mu') = C_1(\mu') + \frac{1}{N_C}\, C_2(\mu'), \tag{16}$$

with $N_C = 3$ the number of colors.

The decay width is given by

$$\begin{aligned} \Gamma =& \frac{G_F^2}{16\pi m_B^2}\, |V_{cb}|^2\, |V_{cs}|^2\, a_1^2\, \frac{\lambda^{1/2}(m_B^2, m_{D^{(*)}}^2, m_{D_{sJ}^{(*)}}^2)}{2m_B} \\ & \times \mathcal{H}_{\alpha\beta}(P_B, P_{D^{(*)}})\, \hat{\mathcal{H}}^{\alpha\beta}(P_{D_{sJ}^{(*)}}), \end{aligned} \tag{17}$$

where $G_F = 1.16637(1) \times 10^{-5}\,\text{GeV}^{-2}$ is the Fermi decay constant [8], $\lambda(a,b,c) = (a+b-c)^2 - 4ab$, V_{cb} and V_{cs} are the corresponding Cabbibo - Kobayashi - Maskawa matrix elements, for which we take $V_{bc} = 0.0413$ and $V_{cs} = 0.974$. P_B, $P_{D^{(*)}}$ and $P_{D_{sJ}^{(*)}}$ are the meson momenta. $\mathcal{H}_{\alpha\beta}(P_B, P_{D^{(*)}})$ is the hadron tensor for the $B^- \to D^{(*)0}$ transition and $\hat{\mathcal{H}}^{\alpha\beta}(P_{D_{sJ}^{(*)}})$ is the hadron tensor for the vacuum $\to D_{sJ}^{(*)-}$ transition. We have

$$\hat{\mathcal{H}}^{\alpha\beta}(P_{D_{sJ}^{(*)}}) = P^\alpha_{D_{sJ}^{(*)}} P^\beta_{D_{sJ}^{(*)}} f^2_{D_{sJ}^{(*)}}, \tag{18}$$

for a scalar or pseudoscalar meson, and

$$\hat{\mathcal{H}}^{\alpha\beta}(P_{D_{sJ}^{(*)}}) = (P^\alpha_{D_{sJ}^{(*)}} P^\beta_{D_{sJ}^{(*)}} - m^2_{D_{sJ}^{(*)}} g^{\alpha\beta}) f^2_{D_{sJ}^{(*)}}, \tag{19}$$

for a vector or an axial-vector meson. In previous equations, $f_{D_{sJ}^{(*)}}$ is the $D_{sJ}^{(*)}$ meson decay constant.

As in Refs. [27, 31], the contraction of hadron tensors, $\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{D^{(*)}})\hat{\mathcal{H}}^{\alpha\beta}(P_{D_{sJ}^{(*)}})$, can be easily written in terms of helicity amplitudes for the $B^- \to D^{(*)}$ transition [9], so that the decay width is given as

$$\begin{aligned} \Gamma =& \frac{G_F^2}{16\pi m_B^2} |V_{cb}|^2 |V_{cs}|^2 a_1^2\, \frac{\lambda^{1/2}(m_B^2, m_{D^{(*)}}^2, m_{D_{sJ}^{(*)}}^2)}{2m_B} \times \\ & \times m^2_{D_{sJ}^{(*)}} f^2_{D_{sJ}^{(*)}} \mathcal{H}_{tt}^{B\to D^{(*)}}(m^2_{D_{sJ}^{(*)}}), \end{aligned} \tag{20}$$

for $D_{sJ}^{(*)}$ a pseudoscalar or scalar meson, and

$$\begin{aligned} \Gamma =& \frac{G_F^2}{16\pi m_B^2} |V_{cb}|^2 |V_{cs}|^2 a_1^2\, \frac{\lambda^{1/2}(m_B^2, m_{D^{(*)}}^2, m_{D_{sJ}^{(*)}}^2)}{2m_B} \times \\ & \times m^2_{D_{sJ}^{(*)}} f^2_{D_{sJ}^{(*)}} \Big[\mathcal{H}_{+1+1}^{B\to D^{(*)}}(m^2_{D_{sJ}^{(*)}}) \\ & + \mathcal{H}_{-1-1}^{B\to D^{(*)}}(m^2_{D_{sJ}^{(*)}}) + \mathcal{H}_{00}^{B\to D^{(*)}}(m^2_{D_{sJ}^{(*)}}) \Big], \end{aligned} \tag{21}$$

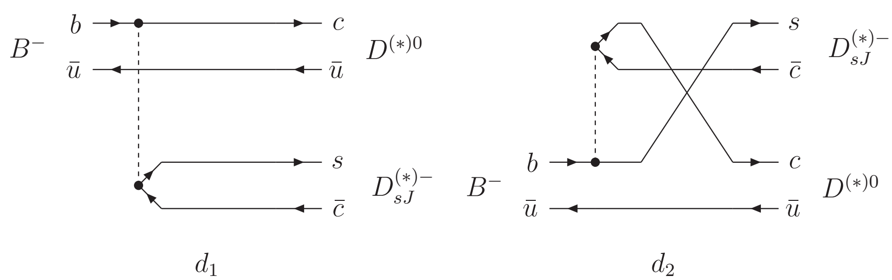

FIG. 1. Schematic representation of B^- decay into $D^{(*)0}D_{sJ}^-$.

for $D_{sJ}^{(*)}$ a vector or axial-vector meson. The different $\mathcal{H}_{rr}$ are defined in Ref. [31] and evaluated in this case at $q^2 = m^2_{D_{sJ}^{(*)}}$. The expressions of the decay constants needed are given in the Appendix.

IV. RESULTS

In this section we present our results for the ratios in Eqs. (1) and (2). We have calculated them using the factorization approximation. As we have already mentioned, the factorization approximation and heavy quark limit predicts ratios of order one for $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons, and negligible for $D_{s1}(2536)$ [15, 33].

¿From Eqs. (20) and (21) we arrive at

$$
\begin{aligned}
R_{D0} =& \frac{\lambda^{1/2}\left(m_B^2, m_D^2, m^2_{D^*_{s0}(2317)}\right) m^2_{D^*_{s0}(2317)} f^2_{D^*_{s0}(2317)} \mathcal{H}_{tt}^{B\to D}\left(m^2_{D^*_{s0}(2317)}\right)}{\lambda^{1/2}(m_B^2, m_D^2, m_{D_s}^2) m_{D_s}^2 f_{D_s}^2 \mathcal{H}_{tt}^{B\to D}(m_{D_s}^2)}, \\
R_{D1} =& \frac{\lambda^{1/2}\left(m_B^2, m_D^2, m^2_{D_{s1}(2460)}\right) m^2_{D_{s1}(2460)} f^2_{D_{s1}(2460)}}{\lambda^{1/2}(m_B^2, m_D^2, m^2_{D_s^*}) m^2_{D_s^*} f^2_{D_s^*}} \times \\
& \times \frac{\left[\mathcal{H}_{+1+1}^{B\to D}\left(m^2_{D_{s1}(2460)}\right) + \mathcal{H}_{-1-1}^{B\to D}\left(m^2_{D_{s1}(2460)}\right) + \mathcal{H}_{00}^{B\to D}\left(m^2_{D_{s1}(2460)}\right)\right]}{\left[\mathcal{H}_{+1+1}^{B\to D}(m^2_{D_s^*}) + \mathcal{H}_{-1-1}^{B\to D}(m^2_{D_s^*}) + \mathcal{H}_{00}^{B\to D}(m^2_{D_s^*})\right]}, \\
R_{D1'} =& \frac{\lambda^{1/2}\left(m_B^2, m_D^2, m^2_{D_{s1}(2536)}\right) m^2_{D_{s1}(2536)} f^2_{D_{s1}(2536)}}{\lambda^{1/2}(m_B^2, m_D^2, m^2_{D_s^*}) m^2_{D_s^*} f^2_{D_s^*}} \times \\
& \times \frac{\left[\mathcal{H}_{+1+1}^{B\to D}\left(m^2_{D_{s1}(2536)}\right) + \mathcal{H}_{-1-1}^{B\to D}\left(m^2_{D_{s1}(2536)}\right) + \mathcal{H}_{00}^{B\to D}\left(m^2_{D_{s1}(2536)}\right)\right]}{\left[\mathcal{H}_{+1+1}^{B\to D}(m^2_{D_s^*}) + \mathcal{H}_{-1-1}^{B\to D}(m^2_{D_s^*}) + \mathcal{H}_{00}^{B\to D}(m^2_{D_s^*})\right]},
\end{aligned}
\tag{22}
$$

and the same for R_{D^*0}, R_{D^*1} and $R_{D^*1'}$ but replacing the meson D by the meson D^*.

Using experimental masses we obtain the ratios

$$
\begin{aligned}
R_{D0} &= 0.9008 \times \left|\frac{f_{D^*_{s0}(2317)}}{f_{D_s}}\right|^2, \\
R_{D^*0} &= 0.7166 \times \left|\frac{f_{D^*_{s0}(2317)}}{f_{D_s}}\right|^2.
\end{aligned}
\tag{23}
$$

The double ratio R_{D^*0}/R_{D0} does not depend on decay constants, and in our model we obtain $R_{D^*0}/R_{D0} = 0.7955$. The experimental value is given by $R_{D^*0}/R_{D0} = 1.50 \pm 0.75$. Our result is small compared to the central experimental value but we are compatible within 1σ. In

the case of the meson $D_{s1}(2460)$ we obtain

$$\begin{aligned} R_{D1} &= 0.7040 \times \left| \frac{f_{D_{s1}(2460)}}{f_{D_s^*}} \right|^2 , \\ R_{D^*1} &= 1.0039 \times \left| \frac{f_{D_{s1}(2460)}}{f_{D_s^*}} \right|^2 , \end{aligned} \tag{24}$$

and for the double ratio R_{D^*1}/R_{D1} we get 1.4260, which agrees well with the experimental result $R_{D^*1}/R_{D1} = 1.32 \pm 0.43$. Finally, for the meson $D_{s1}(2536)$ we obtain

$$\begin{aligned} R_{D1'} &= 0.6370 \times \left| \frac{f_{D_{s1}(2536)}}{f_{D_s^*}} \right|^2 , \\ R_{D^*1'} &= 0.9923 \times \left| \frac{f_{D_{s1}(2536)}}{f_{D_s^*}} \right|^2 , \end{aligned} \tag{25}$$

and for the double ratio $R_{D^*1'}/R_{D1'}$, our value is 1.5578 which in this case is 2σ above the experimental one, 0.90 ± 0.27.

The quality of the experimental results does not allow to be very conclusive as to the goodness of factorization approximation. But one can conclude from Eqs. (23), (24) and (25) that one cannot ignore, as done when using the infinite heavy quark mass limit, phase space and weak matrix element corrections.

The decay constants of pseudoscalar and vector mesons in charmed and charmed-strange sectors are given in Table III. We compare our results with the experimental data and those predicted by different approaches and collected in Refs. [8, 34]. Our original values are those with the symbol (†). The decay constants of vector mesons agree with other approaches. In the case of the pseudoscalar mesons, the decay constants are simply too large. The reason for that is the following: Our CQM presents an OGE potential which has a spin-spin contact hyperfine interaction that is proportional to a Dirac delta function, conveniently regularized, at the origin. The corresponding regularization parameter was fitted to determine the hyperfine splittings between the n^1S_0 and n^3S_1 states in the different flavor sectors, achieving a good agreement in all of them. While most of the physical observables are insensitive to the regularization of this delta term, those related with annihilation processes are affected because these processes are driven by short range operators [35]. The effect is very small in the $^3S_1$ channel as the delta term is repulsive in this case. It is negligible for higher partial waves due to the shielding by the centrifugal barrier. However, it is sizable in the $^1S_0$ channel for which the delta term is attractive.

One should expect the wave functions of the 1^1S_0 and 1^3S_1 states to be very similar [36] except for the very short range. In fact, they are equal if the Dirac delta term is ignored. The values with the symbol (‡) in Table III are referred to the pseudoscalar decay constants which have been calculated using the wave function of the corresponding $^3S_1$ state. We recover the agreement with experiment and also with the predictions of different theoretical approaches. The f_{D_s}/f_D and $f_{D_s^*}/f_{D^*}$ ratios are also shown in the last column of Table III. They are not very sensitive to the delta term and our values agree nicely with experiment and the values obtained in other approaches.

Table IV summarizes the remaining decay constants needed for the calculation we are interested in. There, we show the results from the constituent quark model in which the 1-loop QCD corrections to the OGE potential and the presence of non-$q\bar{q}$ degrees of freedom in $J^P = 1^+$ charmed-strange meson sector are included.

Refs. [37] and [38] calculate the lower bounds of the decay constants of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ analyzing experimental data related to $B \to DD_{sJ}^{(*)}$. Ref. [37] provides the following lower limits:

$$\begin{aligned} |a_1|\, f_{D_{s0}^*(2317)} &= \begin{cases} 58 - 83\,\text{MeV} & \text{from } B^- \text{ decays} \\ 63 - 86\,\text{MeV} & \text{from } \bar{B}^0 \text{ decays} \end{cases} \\ |a_1|\, f_{D_{s1}(2460)} &= \begin{cases} 188^{+40}_{-54}\,\text{MeV} & \text{from } B^- \text{ decays} \\ 152^{+43}_{-62}\,\text{MeV} & \text{from } \bar{B}^0 \text{ decays} \end{cases} \end{aligned} \tag{26}$$

and the authors of Ref. [38] get

$$\begin{aligned} |a_1|\, f_{D_{s0}^*(2317)} &= 74 \pm 11 \\ |a_1|\, f_{D_{s1}(2460)} &= 166 \pm 20 \end{aligned} \tag{27}$$

where the parameter $|a_1| \sim 1$. Our results are compatible with these lower limits.

Our results for the decay constants clearly deviate from the ones obtained in the infinite heavy quark mass limit. In that limit one gets $f_{D_{s0}^*(2317)} = f_{D_s}$, $f_{D_{s1}(2460)} = f_{D_s^*}$ and $f_{D_{s1}(2536)} = 0$, results that lead to a strong disagreement with experiment for the decay width ratios in Eqs. (1) and (2). That was already noticed in Ref. [15], where the authors, using the experimental ratios, estimated that $f_{D_{s0}^*(2317)} \sim \frac{1}{3} f_{D_s}$ and $f_{D_{s0}^*(2317)} \sim f_{D_{s1}(2460)}$ instead. We obtain $f_{D_{s0}^*(2317)}/f_{D_s} = 0.36$, $f_{D_{s0}^*(2317)} \sim 0.72 f_{D_{s1}(2460)}$ and $f_{D_{s1}(2536)} = 59.176\,\text{MeV}$, the latter being small compared to the others but certainly different from zero.

Finally, we show in Table V our results for the ratios written in Eqs. (1) and (2). The symbol (∗) indicates that the ratios have been calculated using the experimental pseudoscalar decay constant in Table III. We get results close to or within the experimental error bars for the $D_{s0}^*(2317)$ meson, which to us is an indication that this meson could be a canonical $c\bar{s}$ state. The incorporation of the non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ channel, enhances the $j_q = 3/2$ component of the $D_{s1}(2536)$ meson and it gives rise to ratios in better agreement with experiment. Note that this state is still an almost pure $q\bar{q}$ state in our description [26].

The situation is more complicated for the $D_{s1}(2460)$ meson. The probability distributions of its $^1P_1$ and $^3P_1$ components are corrected by the inclusion of non-$q\bar{q}$ degrees of freedom, the latter making a $\sim 25\%$ of the wave function. In our calculation, only the pure $q\bar{q}$ component

Approach	f_D (MeV)	f_{D_s} (MeV)	f_{D_s}/f_D
Ours	$297.019^{(\dagger)}$	$416.827^{(\dagger)}$	$1.40^{(\dagger)}$
	$214.613^{(\ddagger)}$	$286.382^{(\ddagger)}$	$1.33^{(\ddagger)}$
Experiment	206.7 ± 8.9	257.5 ± 6.1	1.25 ± 0.06
Lattice (HPQCD+UKQCD)	208 ± 4	241 ± 3	1.162 ± 0.009
Lattice (FNAL+MILC+HPQCD)	217 ± 10	260 ± 10	1.20 ± 0.02
PQL	197 ± 9	244 ± 8	1.24 ± 0.03
QL (QCDSF)	$206 \pm 6 \pm 3 \pm 22$	$220 \pm 6 \pm 5 \pm 11$	$1.07 \pm 0.02 \pm 0.02$
QL (Taiwan)	$235 \pm 8 \pm 14$	$266 \pm 10 \pm 18$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD)	$210 \pm 10^{+17}_{-16}$	$236 \pm 8^{+17}_{-14}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL	$211 \pm 14^{+2}_{-12}$	$231 \pm 12^{+6}_{-1}$	1.10 ± 0.02
QCD Sum Rules	177 ± 21	205 ± 22	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules	203 ± 20	235 ± 24	1.15 ± 0.04
Field Correlators	210 ± 10	260 ± 10	1.24 ± 0.03
Light Front	206	268.3 ± 19.1	1.30 ± 0.04
Approach	f_{D^*} **(MeV)**	$f_{D_s^*}$ **(MeV)**	$f_{D_s^*}/f_{D^*}$
Ours	$247.865^{(\dagger)}$	$329.441^{(\dagger)}$	$1.33^{(\dagger)}$
RBS	340 ± 22	375 ± 24	1.10 ± 0.06
RQM	315	335	1.06
QL (Italy)	234	254	$1.04 \pm 0.01^{+2}_{-4}$
QL (UKQCD)	$245 \pm 20^{+0}_{-2}$	$272 \pm 16^{+0}_{-20}$	1.11 ± 0.03
BS	237	242	1.02
RM	262 ± 10	298 ± 11	1.14 ± 0.09

TABLE III. Theoretical predictions of decay constants for pseudoscalar and vector charmed mesons. The data have been taken from Ref. [8] for pseudoscalar mesons and from Ref. [34] for vector mesons. PQL≡Partially-Quenched Lattice calculation, QL≡Quenched Lattice calculations, RBS≡Relativistic Bethe-Salpeter, RQM≡Relativistic Quark Model, BS≡Bethe-Salpeter Method and RM≡Relativistic Mock meson model.

Meson	f_D (MeV)	$\sqrt{M_D} f_D$ (GeV$^{3/2}$)
$D_{s0}^*(2317)$	118.706	0.181
$D_{s1}(2460)$	165.097	0.259
$D_{s1}(2536)$	59.176	0.094

TABLE IV. Decay constants calculated within the CQM including 1-loop QCD corrections to the OGE potential and non-$q\bar{q}$ structure in channel 1^+.

of the $D_{s1}(2460)$ meson has been used to evaluate the $\Gamma(B \to D^{(*)} D_{s1}(2460))$ decay width. The values we get for the corresponding ratios in Eqs. (1) are lower than experiment.

V. CONCLUSIONS

We have performed a calculation of the ratios shown in Eqs. (1) and (2) working within the framework of the constituent quark model and in the factorization approximation. These ratios have been recently reported by the Belle Collaboration in Ref. [7] and provide important information to check the structure of the $D_{s0}^*(2317)$, $D_{s1}(2460)$ and $D_{s1}(2536)$ mesons.

The strong disagreement found between the heavy quark limit predictions and the experimental data motivates the introduction of the finite c-quark mass effects, which is done easily in the context of the constituent quark model.

In the heavy quark limit and in the factorization approximation, the ratios should be of order one for the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons and very small for $D_{s1}(2536)$. In contrast, the experimental data indicate that the decay pattern of the $D_{s1}(2536)$ follows the expectations, but that is not the case for the $D_{s0}^*(2317)$ and $D_{s1}(2460)$.

The mass of the $D_{s0}^*(2317)$ meson is lowered towards the experimental value with the inclusion of the 1-loop corrections to the OGE potential. We also obtain ratios compatible with the experimental data. Our results indicate that this meson could be described as a canonical $c\bar{s}$ state.

We incorporate the non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ channel. The $D_{s1}(2536)$ meson remains almost a pure $q\bar{q}$ state and its $j_q = 3/2$ component is enhanced. This gives the correct ratios for the $D_{s1}(2536)$ meson. Together with the properties calculated in Refs. [26, 27], this is to us evidence of a good description of the $D_{s1}(2536)$ meson.

The $D_{s1}(2460)$ has a sizable non-$q\bar{q}$ component which contributes to the decays under study. This contribution has not been calculated, as goes beyond the scope of this work. We have computed the ratios considering only the contribution coming from the $q\bar{q}$ structure of

	$X \equiv D^*_{s0}(2317)$		$X \equiv D_{s1}(2460)$		$X \equiv D_{s1}(2536)$	
	The.	Exp.	The.	Exp.	The.	Exp.
$\mathcal{B}(B \to DX)/\mathcal{B}(B \to DD_s)$	$0.19^{(*)}$	0.10 ± 0.03	-	-	-	-
$\mathcal{B}(B \to D^*X)/\mathcal{B}(B \to D^*D_s)$	$0.15^{(*)}$	0.15 ± 0.06	-	-	-	-
$\mathcal{B}(B \to DX)/\mathcal{B}(B \to DD^*_s)$	-	-	0.177	0.44 ± 0.11	0.021	0.049 ± 0.010
$\mathcal{B}(B \to D^*X)/\mathcal{B}(B \to D^*D^*_s)$	-	-	0.252	0.58 ± 0.12	0.032	0.044 ± 0.010

TABLE V. Ratios of branching fractions for nonleptonic decays $B \to D^{(*)}D^{(*)}_{sJ}$.

the $D_{s1}(2460)$ meson. The ratios are a factor 2 below the experimental ones.

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Appendix A: DECAY CONSTANTS

In our model, and due to the normalization of our nonrelativistic meson states, the decay constants are given by [39]

$$\begin{aligned}
f_{M(0^-)} &= -i\sqrt{\frac{2}{m_{M(0^-)}}}\,\langle 0|J^{f_2f_1}_{A0}(0)|M(0^-),\vec{0}\rangle\,,\\
f_{M(0^+)} &= +i\sqrt{\frac{2}{m_{M(0^+)}}}\,\langle 0|J^{f_2f_1}_{V0}(0)|M(0^+),\vec{0}\rangle\,,\\
f_{M(1^-)} &= -\sqrt{\frac{2}{m_{M(1^-)}}}\,\langle 0|J^{f_2f_1}_{V3}(0)|M(1^-),0\,\vec{0}\rangle\,,\\
f_{M(1^+)} &= +\sqrt{\frac{2}{m_{M(1^+)}}}\,\langle 0|J^{f_2f_1}_{A3}(0)|M(1^+),0\,\vec{0}\rangle\,,
\end{aligned} \tag{A1}$$

for pseudoscalar, scalar, vector and axial-vector mesons, respectively. $J^{f_2f_1}_{V\mu}$ and $J^{f_2f_1}_{A\mu}$ are the vector and axial-vector charged weak current operators for a specific pair of quark flavors f_1 and f_2.

The corresponding matrix elements are given by

$$\begin{aligned}
\langle 0|J^{f_2f_1}_{A0}(0)|M(0^-),\vec{0}\rangle =& i\frac{\sqrt{3}}{\pi}\int d|\vec{p}|\;|\vec{p}|^2\hat{\phi}^{(M(0^-))}_{f_1,f_2}(|\vec{p}|)\sqrt{\frac{(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(-\vec{p})E_{f_2}(\vec{p})}}\\
&\times\left[1-\frac{|\vec{p}|^2}{(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}\right],
\end{aligned}$$

$$
\begin{aligned}
\langle 0|J_{V0}^{f_2f_1}(0)|M(0^+),\vec{0}\rangle =& i\frac{\sqrt{3}}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}_{f_1,f_2}^{(M(0^+))}(|\vec{p}|)\sqrt{\frac{(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(-\vec{p})E_{f_2}(\vec{p})}}\\
&\times\left[\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}-\frac{1}{E_{f_1}(-\vec{p})+m_{f_1}}\right],\\
\langle 0|J_{V3}^{f_2f_1}(0)|M(1^-,L=0),0\,\vec{0}\rangle =& -\frac{\sqrt{3}}{\pi}\int d|\vec{p}|\,|\vec{p}|^2\hat{\phi}_{f_1,f_2}^{(M(1^-,L=0))}(|\vec{p}|)\sqrt{\frac{(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(-\vec{p})E_{f_2}(\vec{p})}}\\
&\times\left(1+\frac{|\vec{p}|^2}{3(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}\right),\\
\langle 0|J_{V3}^{f_2f_1}(0)|M(1^-,L=2,0)\,\vec{0}\rangle =& -\frac{2}{\pi}\sqrt{\frac{2}{3}}\int d|\vec{p}|\,\hat{\phi}_{f_1,f_2}^{(M(1^-,L=2))}(|\vec{p}|)\sqrt{\frac{(E_{f_2}(\vec{p})+m_{f_2})(E_{f_1}(-\vec{p})+m_{f_1})}{4E_{f_1}(-\vec{p})E_{f_2}(\vec{p})}}\\
&\times\frac{|\vec{p}|^4}{(E_{f_2}(\vec{p})+m_{f_2})(E_{f_1}(-\vec{p})+m_{f_1})},\\
\langle 0|J_{A3}^{f_2f_1}(0)|M(1^+,S=0),0\,\vec{0}\rangle =& \frac{1}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}_{f_1,f_2}^{(M(1^+,S=0))}(|\vec{p}|)\sqrt{\frac{(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(-\vec{p})E_{f_2}(\vec{p})}}\\
&\times\left[\frac{1}{E_{f_1}(-\vec{p})+m_{f_1}}-\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}\right],\\
\langle 0|J_{A3}^{f_2f_1}(0)|M(1^+,S=1),0\,\vec{0}\rangle =& -\frac{\sqrt{2}}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}_{f_1,f_2}^{(M(1^+,S=1))}(|\vec{p}|)\sqrt{\frac{(E_{f_1}(-\vec{p})+m_{f_1})(E_{f_2}(\vec{p})+m_{f_2})}{4E_{f_1}(-\vec{p})E_{f_2}(\vec{p})}}\\
&\times\left[\frac{1}{E_{f_1}(-\vec{p})+m_{f_1}}+\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}\right],
\end{aligned}
\tag{A2}
$$

where E_{f_1} and E_{f_2} are the relativistic energy of the quarks. For 0^- and 0^+ we have only one possible contribution. In the case of the $J^P=1^-$ meson we have two contributions coming from the two possible values of the relative angular momentum. For $J^P=1^+$ states and since C-parity is not well defined in charmed and charmed-strange mesons the wave function is a mixture of ${}^1P_1$ and ${}^3P_1$ partial waves and thus there are also two contributions.

1. Decay constants in the heavy quark limit

For the low-lying positive parity excitations, any quark model predicts four states that in the ${}^{2S+1}L_J$ basis correspond to ${}^1P_1$, ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$. As charge conjugation is not well defined in the heavy-light sector, ${}^1P_1$ and ${}^3P_1$ states can mix under the interaction.

In the infinite heavy quark mass limit, heavy quark symmetry (HQS) predicts two degenerate P-wave meson doublets, labeled by $j_q=1/2$ with $J^P=0^+,1^+$ ($|1/2,0^+\rangle,|1/2,1^+\rangle$) and $j_q=3/2$ with $J^P=1^+,2^+$ ($|3/2,1^+\rangle,|3/2,2^+\rangle$). In this limit, the meson properties are governed by the dynamics of the light quark, which is characterized by its total angular momentum $j_q=s_q+L$, where s_q is the light quark spin and L the orbital angular momentum. The total angular momentum of the meson J is obtained coupling j_q to the heavy quark spin, s_Q.

A change of basis allows to express the above states in terms of the ${}^{2S+1}L_J$ basis, by recoupling angular momenta, as

$$
\begin{aligned}
|1/2,0^+\rangle &= +|{}^3P_0\rangle\,,\\
|1/2,1^+\rangle &= +\sqrt{\frac{1}{3}}|{}^1P_1\rangle+\sqrt{\frac{2}{3}}|{}^3P_1\rangle\,,\\
|3/2,1^+\rangle &= -\sqrt{\frac{2}{3}}|{}^1P_1\rangle+\sqrt{\frac{1}{3}}|{}^3P_1\rangle\,,\\
|3/2,2^+\rangle &= +|{}^3P_2\rangle\,,
\end{aligned}
\tag{A3}
$$

where in the ${}^{2S+1}L_J$ wave functions we couple heavy and light quark spins, in this order, to total spin S.

	$D_0^*(2400)$	$D_1(2420)$	$D_1(2430)$	$D_2^*(2460)$
$^3P_0$	$+$, 1.0000	-	-	-
$^1P_1$	-	$-$, 0.5903	$-$, 0.4097	-
$^3P_1$	-	$+$, 0.4097	$-$, 0.5903	-
$^3P_2$	-	-	-	$+$, 0.99993
$1/2, 0^+$	$+$, 1.0000	-	-	-
$1/2, 1^+$	-	$+$, 0.0063	$-$, 0.9937	-
$3/2, 1^+$	-	$+$, 0.9937	$+$, 0.0063	-
$3/2, 2^+$	-	-	-	$+$, 0.99993
	$D_{s0}^*(2317)$	$D_{s1}(2536)$	$D_{s1}(2460)$	$D_{s2}^*(2573)$
	D_{s0}^*	D_{s1}	D'_{s1}	D_{s2}^*
$^3P_0$	$+$, 1.0000	-	-	-
$^1P_1$	-	$-$, 0.7210	$-$, 0.1880	-
$^3P_1$	-	$+$, 0.2770	$-$, 0.5570	-
$^3P_2$	-	-	-	$+$, 0.99991
$1/2, 0^+$	$+$, 1.0000	-	-	-
$1/2, 1^+$	-	$-$, 0.0038	$-$, 0.7390	-
$3/2, 1^+$	-	$+$, 0.9942	$-$, 0.0060	-
$3/2, 2^+$	-	-	-	$+$, 0.99991

TABLE VI. Probability distributions and their relative phases for the four states predicted by CQM in the two basis described in the text. In the 1^+ strange sector the effects of non-$q\bar{q}$ components are included, see text for details.

In the actual calculation the ideal mixing in Eq. (A3) between $^1P_1$ and $^3P_1$ states changes due to finite charm quark mass effects. Our CQM model predicts the mixed states shown in Table VI, which are very similar to the HQS states. This is expected since the c-quark is much heavier ($m_c = 1763\,\mathrm{MeV}$) than the light ($m_n = 313\,\mathrm{MeV}$) or strange ($m_s = 555\,\mathrm{MeV}$) quarks. Note that we also have mixing, even if small, between the $^3P_2$ and $^3F_2$ partial waves in 2^+ mesons. This is due to the OGE tensor term.

In Ref. [26] we have studied the $J^P = 1^+$ charmed-strange mesons, finding that the $J^P = 1^+$ $D_{s1}(2460)$ has an important non-$q\bar{q}$ contribution whereas the $J^P = 1^+$ $D_{s1}(2536)$ is almost a pure $q\bar{q}$ state. The presence of non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ charmed-strange meson sector enhances the $j_q = 3/2$ component of the $D_{s1}(2536)$. This wave function explains most of the experimental data, as shown in Refs. [26, 27]. For this sector only the $q\bar{q}$ probabilities are given in Table VI.

Here, we want to recover the results of the decay constants predicted by HQS. ¿From Eqs. (A2), we only need to do the limit $m_{f_1} \to \infty$

- Decay constants of D_s and D_s^* in the heavy quark limit:

$$
\begin{aligned}
f_{D_s} &\xrightarrow{m_{f_1}\to\infty} \frac{\sqrt{3}}{\pi}\sqrt{\frac{1}{m_{D_s}}}\int d|\vec{p}|\;|\vec{p}|^2\hat{\phi}_{f_1,f_2}^{(D_s,{}^1S_0)}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\;, \\
f_{D_s^*} &\xrightarrow{m_{f_1}\to\infty} \frac{\sqrt{3}}{\pi}\sqrt{\frac{1}{m_{D_s^*}}}\int d|\vec{p}|\,|\vec{p}|^2\hat{\phi}_{f_1,f_2}^{(D_s^*,{}^3S_1)}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\;.
\end{aligned}
\tag{A4}
$$

The D_s and D_s^* mesons belong to the doublet $(|1/2,0^-\rangle, |1/2,1^-\rangle)$ predicted by HQS. In such a limit the mesons are degenerate and the radial wave functions of the $^1S_0$ and $^3S_1$ components are the same. Therefore, we obtain $f_{D_s} = f_{D_s^*}$ in that limit.

- Decay constants of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ in the heavy quark limit:

$$f_{D^*_{s0}(2317)} \xrightarrow{m_{f_1}\to\infty} -\frac{\sqrt{3}}{\pi}\sqrt{\frac{1}{m_{D^*_{s0}(2317)}}}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}^{(D^*_{s0}(2317),{}^3P_0)}_{f_1,f_2}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\,\frac{1}{E_{f_2}(\vec{p})+m_{f_2}},$$

$$\begin{aligned} f_{D_{s1}(2460)} \xrightarrow{m_{f_1}\to\infty} & +\sqrt{\frac{2}{m_{D_{s1}(2460)}}}\times \\ & \times\left\{-\sqrt{\frac{1}{6}}\frac{1}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}^{(D_{s1}(2460),{}^1P_1)}_{f_1,f_2}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\,\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}\right. \\ & \left.-\sqrt{\frac{2}{3}}\frac{1}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}^{(D_{s1}(2460),{}^3P_1)}_{f_1,f_2}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\,\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}\right\}. \end{aligned} \tag{A5}$$

The $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons belong to the doublet $(|1/2,0^+\rangle, |1/2,1^+\rangle)$ predicted by HQS. Again, in such a limit, the mesons are degenerate and the radial wave functions of the ${}^3P_0$, ${}^1P_1$ and ${}^3P_1$ components would be equal. Therefore, we would obtain $f_{D^*_{s0}(2317)} = f_{D_{s1}(2460)}$ in that limit.

- Decay constant of $D_{s1}(2536)$ in the HQS:

$$\begin{aligned} f_{M(1^+,D_{s1}(2536))} \xrightarrow{m_{f_1}\to\infty} & +\sqrt{\frac{2}{m_{D_{s1}(2536)}}}\times \\ & \times\left\{+\sqrt{\frac{1}{3}}\frac{1}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}^{(D_{s1}(2536),{}^1P_1)}_{f_1,f_2}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\,\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}\right. \\ & \left.-\sqrt{\frac{1}{3}}\frac{1}{\pi}\int d|\vec{p}|\,|\vec{p}|^3\hat{\phi}^{(D_{s1}(2536),{}^3P_1)}_{f_1,f_2}(|\vec{p}|)\sqrt{1+\frac{m_{f_2}}{E_{f_2}(\vec{p})}}\,\frac{1}{E_{f_2}(\vec{p})+m_{f_2}}\right\}. \end{aligned} \tag{A6}$$

Therefore, considering that the radial wave functions of the ${}^1P_1$ and ${}^3P_1$ components are the same, $f_{D_{s1}(2536)}$ is equal zero in the heavy quark limit.

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